DEPARTMENT OF COMMERCE

TECHNOLOGIC PAPERS

OF THE

BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

No. 112

STANDARDIZATION OF THE SAYBOLT UNIVERSAL VISCOSIMETER

BY

WINSLOW H. HERSCHEL, Associate Physicist
Bureau of Standards

[2d Edition]
ISSUED APRIL 30, 1919

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1. INTRODUCTION

The Saybolt Universal viscosimeter has always been at a disadvantage, as compared with the Engler instrument, because it had never been standardized with respect to its principal dimensions. To overcome this difficulty the Bureau of Standards entered into negotiations with George M. Saybolt, and he agreed to accept certain instruments as standard.

In a previous paper the method was described for determining an equation showing the relation between readings of certain Saybolt Universal viscosimeters and the absolute viscosity. The object of the present investigation has been to determine a similar equation for instruments of standard dimensions. As the methods employed have been mainly the same as before,

frequent references to Technologic Paper No. 100 will be necessary, although it will, in general, be assumed that the reader is familiar with the efflux method of determining viscosity, as described in that paper.

2. THE NEED OF STANDARDIZATION

In calculations involving viscosity such units as Saybolt seconds and Engler degrees can not be used. It is necessary to express viscosity by an absolute unit whose value is independent of the instrument by which it is obtained. To completely standardize a viscosimeter, therefore, it is necessary to adopt normal dimensions, with allowable tolerances, and to determine an equation for converting viscosimeter readings into absolute viscosity.

For measuring the viscosity of oils the two most commonly used efflux viscosimeters are the Saybolt Universal and the Engler. In both instruments there is a cylindrical container for the liquid whose viscosity is to be measured, an outlet tube at the bottom of the container, and a bath surrounding the container to control the temperature. The instruments differ in the dimensions of the various parts and in the method of starting the flow. The essential dimensions of the Saybolt instrument have been published by Gill[1] and by Meissner,[2] but they are not in good agreement, so that all equations previously determined must be regarded as applying to certain instruments, but not to instruments of standard dimensions.

3. THE STANDARD DIMENSIONS OF THE ENGLER VISCOSIMETER

The Engler instrument has been standardized with respect to its principal dimensions, although, as has been indicated elsewhere,[3] the much-used Ubbelohde equation is inaccurate. As the methods of standardizing the Engler viscosimeter apply for the most part to the Saybolt instrument also, it will be of interest to consider them briefly. The Engler instrument was originally described by its designer,[4] and on April 1, 1907, the principal German testing laboratories adopted the normal dimensions and tolerances as given in Table 1.[5]

---

[1] A. H. Gill, Oil Analysis, p. 29; 1913.

[2] W. Meissner, Chemische Revue über die Fett- und Harz-Industrie, 19, p. 9; 1912.

[3] Winslow H. Herschel, Technologic Paper No. 100, Bureau of Standards, p. 27; 1917. (This paper will be referred to subsequently simply as T. P. 100.)

[4] C. Engler, Zeitschrift für angewandte Chemie, 5, p. 725; 1892.

[5] See Engler-Hofer, Das Erdöl, 4, p. 99, 1913; Chem. Ztg., 31, p. 447, 1907; Chem. Revue über die Fett- und Harz-Industrie, 14, p. 118, 1907; Holde-Mueller, The Examination of Hydrocarbon Oils, p. 105, 1915. (The obsolete reference to 240 cc may still be found in many books.)

TABLE 1.—Standard Dimensions of Engler Viscosimeter

Specified distances	Normal dimensions	Tolerance
	Cm.	Cm.
Length of outlet tube, l	2.0	±0.01
Inside diameter of outlet tube, at upper end, d_1	.29	± .002
Inside diameter of outlet tube, at lower end, d_2	.28	± .002
Outside diameter of outlet tube, d_3	.45	± .02
Length of outlet tube, projecting below bottom of the bath, l_1	.30	± .03
Inside diameter of container, D	10.6	± .10
Depth of cylindrical part of container below the gage points, h_3	2.5	± .10
Height of the gage points above lower end of outlet tube= initial head, h_1	5.2	± .05

It was originally directed to measure out 240 cm^3 of liquid in a flask and to pour into the container, but it is evident that the level of the liquid at the start, rather than the volume in the container, is the factor which controls the pressure which in turn produces the flow. For the volume of the dished part of the container—that is, the capacity of the container below the gage points—in excess of the 200 cm^3 to be withdrawn, can have no effect upon the head causing flow. Moreover, since the contour of the bottom of the container is not specified, it might be expected to differ in different instruments and cause a consequent variation in the volume contained in the dished part.

The directions finally adopted are to pour in an excess of liquid so as to cover the gage points, and then adjust the height of the liquid to the level of the gage points by removing the excess with a pipette. The time of discharge for water at 20° C (68° F), usually called the water rate, must lie between 50 and 52 seconds. Meissner [6] says that it is 50.94 seconds for an instrument of normal dimensions.

4. THE MEASUREMENT OF THE DIMENSIONS OF THE SAYBOLT UNIVERSAL VISCOSIMETER

It has long been realized that the Saybolt instrument would remain at a great disadvantage in competition with the Engler, until it was standardized by the adoption of normal dimensions and suitable tolerances similar to those given in Table 1, for the Engler instrument. As a result of negotiations between George M. Saybolt and this Bureau, Mr. Saybolt sent four viscosimeters, Nos. 108 to 111, to be measured and to serve as a basis for the adoption of normal dimensions and tolerances.

[6] W. Meissner, Chem. Revue über die Fett- und Harz-Industrie, 17, p. 202; 1910.

As the time of discharge, which is used as a measure of the viscosity, varies very rapidly with the diameter of the outlet tube, it was necessary that this dimension should be measured with great accuracy.

(*a*) MEISSNER'S DETERMINATIONS.—Meissner's methods of measurement are suggestive and will be briefly described. The mean diameter of the outlet tube was determined as follows:

> First, cylindrical rods were turned to exactly fit the tube. Secondly, the diameter was measured at the upper and lower ends by means of a special plug gage of circular section, with a 1 : 40 taper. The mean was taken of these three measurements. The value of d_m is nevertheless subject to considerable uncertainty since the workmanship of the tube * * * is not precise. * * * d varied between 1.77 and 1.79 mm.

The length of the outlet tube, l, and the initial head, h_1, were determined with the help of a rod turned small enough to pass easily through the outlet tube as far as a shoulder. To determine l the rod was put through the tube from above, and the length of rod projecting below the bottom of the tube was measured with a vernier depth gage. To find h_1 the rod was put through the tube from below, and the depth gage was used to measure the distance from the upper end of the rod to the overflow rim which fixes the initial level of the liquid.

(*b*) MEASUREMENTS BY THE BUREAU OF STANDARDS.—The dimensions of Saybolt viscosimeters Nos. 108 to 111 were determined without the use of special gages. The inside diameters of the outlet tubes were measured with a micrometer microscope. In finding the diameter at the middle of the tubes it was found convenient to focus on the points of inside calipers made by bending a wire in the form of a V or Y, although the calipers could often be dispensed with by focusing on particles of dust. The length of tube was found by subtracting the distance from the overflow rim to the upper end of the outlet tube from the overall distance from the rim to the lower end of the tube. The final results of the measurements of these viscosimeters are given in Table 2. The symbols for the dimensions are the same as in Table 1.

TABLE 2.—Dimensions of Saybolt Universal Viscosimeters

Dimension	Values as measured by Bureau of Standards			
	No. 108	No. 109	No. 110	No. 111
l ..... centimeters..	1.217	1.233	1.235	1.230
d average .....do....	.1781	.1764	.1759	.1770
d_3 outside .....do....	.30	.31	.30	.28
D .....do....	2.975	2.972	2.974	2.974
h_1 .....do....	12.448	12.542	12.520	12.551
h_2 .....do....	3.816	3.893	3.882	3.913
h average .....do....	7.300	7.393	7.377	7.411
Capacity of container..... cubic centimeters..	69.4	69.5	69.8	69.6

Table 3 gives the normal dimensions and allowable variations proposed by the Bureau of Standards and accepted by Mr. Saybolt on October 1, 1917. It will be seen that the normal dimensions are practically the same as the average of corresponding dimensions in Table 2. The tolerances were determined by a compromise between two considerations. Too large a tolerance would cause too great a variation in efflux time between different instruments, while too small a tolerance would unnecessarily increase the cost of manufacture in attempting to obtain a degree of accuracy not required in technical work. For precise scientific work, some form of viscosimeter should be used which is provided with an outlet tube of much greater length as compared with its diameter.[7]

TABLE 3.—Dimensions of the Standard Saybolt Universal Viscosimeter

Dimension	Minimum	Normal	Maximum
	Cm	Cm	Cm
Diameter of outlet tube, d	0.1750	0.1765	0.1780
Length of outlet tube, l	1.215	1.225	1.235
Outer diameter of outlet tube, at lower end, d_3	.28	.30	.32
Height of overflow rim above bottom of outlet tube, h_1	12.40	12.50	12.60
Diameter of container, D	2.955	2.975	2.995
Average head, h (calculated)	7.16	7.36	7.56

[7] T. P. 100, p. 12.

5. THE EFFECT OF VARIATIONS IN DIMENSIONS OF MINOR IMPORTANCE

The Saybolt viscosimeter is so constructed that the lower end of the outlet tube is about 1.0 cm above the bottom of the bath, the exact distance depending upon the thickness of washers in compression. This distance, therefore, can not be specified, as with the Engler instrument where the bath and container are rigidly soldered together. If, in the Engler instrument, l_1 were too short, the liquid might spread out on the bottom of the bath, thus increasing the effect of surface tension, while if l_1 were too long, the temperature of the liquid in the outlet tube might be perceptibly lower than the temperature of the liquid in the container. These difficulties are avoided by the design of the Saybolt instrument.

Of the dimensions in Table 3, the least important is the outer diameter of the outlet tube. This determines the resistance to flow due to surface tension, which must be considered because the discharge is not submerged. The tolerance for this dimension has been taken as ± 0.02 cm, the same as given in Table 1 for the Engler viscosimeter.

Meissner's formula for calculating the average head may be written

$$\text{Average head, } h = \frac{h_1 - h_2}{\log_e\left(\frac{h_1}{h_2}\right)} \qquad (1)$$

where h_1 and h_2 are the initial and final heads, respectively. This formula is unsatisfactory when the velocity of flow is high,[8] but it is the best available and was used to calculate the average head in Tables 2 and 3.

The diameter of the container, D, is of importance because it enters indirectly into equation (1), being necessary to obtain h_2 by the formula

$$\frac{\pi}{4} D^2 (h_1 - h_2) = 60, \qquad (2)$$

60 cm^3 being the volume discharged in time, t. The so-called Saybolt viscosity is the time, t, expressed in seconds. It follows from equation (2) that when D has the minimum value of 2.955 cm, $h_1 - h_2$ is equal to 8.75 cm. Therefore the cylindrical part of the container must have at least this height. Meissner found it to be 10.35 cm, while it is about 9.7 cm in the standard Saybolt Universal viscosimeter.

[8] T. P. 100, p. 40.

6. EQUATION FOR DETERMINING VISCOSITY FROM DIMENSIONS OF EFFLUX INSTRUMENT AND THE TIME OF DISCHARGE

Efflux instruments, such as the Saybolt Universal and the Engler viscosimeters, may be used to determine kinematic viscosity, which is the name given to the ratio of absolute viscosity to the density. If absolute viscosity is required, it can be obtained by multiplying the kinematic viscosity by the density, which must be determined by an auxiliary instrument.

Kinematic viscosity may be calculated by the equation [9]

$$\frac{\mu}{\gamma}=\frac{\pi\, g d^4 t}{128\, Q\, (l+\lambda)}\left(h-\frac{mv^2}{g}\right) \tag{3}$$

where μ is in poises, the cgs unit of viscosity, γ is the density in grams per cubic centimeter, d and l are the diameter and length, respectively, of the outlet tube, in centimeters; Q is the volume in cubic centimeters, discharged in the time, t, in seconds; λ is the "Couette correction," which must be added to the measured length of tube to get the effective length; h is the average head, in centimeters, of liquid of density, γ; and m is the coefficient of the kinetic energy correction, $\frac{m\,v^2}{g}$, g being the acceleration due to gravity, or 981 cm per second, per second, and v the mean velocity in centimeters per second.

Table 3 gives all values needed for finding the kinematic viscosity from equation (3) with the exception of λ and m. Unfortunately, the value of 1.12 for m, which is commonly used with long-tube viscosimeters, [10] can not be used for the standard Saybolt Universal viscosimeter where the ratio of length to diameter of the outlet tube is only 6.94. The Couette correction is also unknown, because it can have neither the value of zero, which is generally employed with long-tube viscosimeters, nor can it be equal to 0.82 d, which, according to Higgins,[11] is its value in the case of submerged discharge.

If equation (3) is written in the form

$$\frac{\mu}{\gamma}=A\, t-\frac{B}{t}, \tag{4}$$

it is evident that A and B are instrumental constants which may be obtained by finding the time of discharge for two liquids

[9] T. P. 100, p. 6.
[10] E. C. Bingham and R. F. Jackson, Scientific Paper No. 298, Bureau of Standards, p. 65; 1917.
[11] W. F. Higgins, Collected Researches, Nat. Phys. Lab., 11, p. 9; 1914.

having, preferably, a considerable difference in viscosity. Values of the unknown quantities in equation (3) may then be calculated by combining equations (3) and (4). It was found in this way that for Saybolt instruments Nos. 580 and 727, the Couette correction had a value of 0.4 d, while m was equal to 0.97.[12] These instruments were not of standard dimensions and it is not known upon what the values of λ and m depend, but presumably these constants would be about the same for a standard Saybolt Universal viscosimeter, as for instruments Nos. 580 and 727.

7. HIGGINS'S METHOD FOR DETERMINING THE RELATION BETWEEN KINEMATIC VISCOSITY AND TIME OF DISCHARGE

As indicated by Higgins, if both sides of equation (4) are divided by t, and values of $\frac{\mu}{\gamma t}$ as ordinates are plotted against values of $\frac{1}{t^2}$, a straight line will be obtained if A and B are constants. The value of A may be read on the scale of ordinates at the point where the calibration curve intersects the axis of ordinates, and the tangent of the acute angle between the extended straight portion of the calibration curve and the axis of abscissas gives the value of B.

Table 4 shows times of discharge obtained with the four Saybolt instruments, each time, both in this table and in Table 5, being the average for 10 runs. With the exception of olive oil, which has the highest viscosity of any liquid listed in Table 4, and the 60 per cent sugar solutions which also had a high viscosity, the difference between the maximum and the minimum time, for any of the four instruments, exceeded 2 per cent only twice out of 24 tests, both cases being for water at 20° C (68° F).

Fig. 1 has been plotted from data of Tables 4 and 5. The bend in the calibration curve indicates that the flow is turbulent when water is used, so that A and B have different values than for viscous flow. Therefore water is not a suitable calibrating liquid for these viscosimeters. Fig. 1 also shows a disagreement between tests made with solutions of ethyl alcohol, sucrose, and glycerol. The kinematic viscosities of glycerol solutions were calculated from data of Archbutt and Deeley,[13] while the kinematic viscosities of distilled water, ethyl alcohol solutions, and sucrose solutions

[12] T. P. 100, p. 30; Proc., A. S. T. M., 17 (II), p. 562; 1917.
[13] L. Archbutt, and R. M. Deeley, Lubrication and Lubricants, p. 161; 1912.

were taken from data of Bingham and Jackson.[14] As pointed out by Upton [15] there is a doubt as to the accuracy of Archbutt and Deeley's values for glycerol solutions. Other reasons for discarding the tests with these solutions are considered in the Appendix.

TABLE 4.—Comparison of Four Standard Saybolt Viscosimeters

LIQUIDS OF KNOWN VISCOSITY

Liquid	Temperature	Time of discharge for instrument No.—			
		108	109	110	111
	°C	Sec.	Sec.	Sec.	Sec.
Water	15	30.82	31.34	31.42	31.22
Do	20	30.90	30.78	30.10	30.88
Do	20	30.72	31.02	30.12	30.28
Do	50	28.76	28.76	28.56	28.68
Do	75	28.22	28.06	28.04	28.06
10 per cent ethyl alcohol	15	33.50	33.16	33.00	33.48
Do	20	32.06	32.12	31.98	31.96
30 per cent ethyl alcohol	20	35.24	35.24	35.10	35.24
Do	20	35.68	35.64	35.82	35.74
50 per cent ethyl alcohol	Zero	.....	.....	.....	53.34
Do	15	38.88	38.90	39.12	38.98
Do	15	39.40	39.50	39.08	38.92
Do	20	36.74	36.76	36.52	36.54
20 per cent sucrose, old	20	34.16	34.12	33.88	34.06
20 per cent sucrose, fresh	20	33.62	33.68	33.70	33.58
40 per cent sucrose, old	20	45.26	45.56	44.78	45.08
40 per cent sucrose, fresh	20	44.66	45.10	44.52	44.64
60 per cent sucrose, old	20	186.68	190.52	193.66	194.38
60 per cent sucrose, fresh	20	217.50	213.30	212.92	214.18

LIQUIDS OF UNKNOWN VISCOSITY

Olive oil	± 22	478.80	481.80	475.26	461.30
Three-fourths spindle oil One-fourth kerosene	.....	58.10	58.30	58.04	58.22
Press oil No. 3	15	54.66	53.86	53.62	54.16
Do	20	50.28	49.86	49.76	49.80
Do	25	48.54	48.18	48.38	48.24
Meter oil	20	65.78	65.50	65.58	65.52
Press oil No. 2	25	97.14	97.46	95.38	96.04
Press oil No. 4	25	115.32	113.98	114.74	114.98
Press oil No. 1	25	171.38	171.58	171.88	170.80

[14] Kinematic viscosities of these liquids, calculated from data of Bingham and Jackson and other experimenters, are given in the Appendix of T. P. 100, p. 52.

[15] G. E. Upton, Proc., A. S. T. M., 15 (1), p. 323; 1915.

TABLE 5.—Comparison of Two Standard Saybolt Universal Viscosimeters with Engler Instrument No. 2204 U

Liquid	Temperature (approximate)	Time of discharge			Kinematic viscosity from Engler instrument
		Saybolt No.—		Engler[a]	
		110	111		
	°C	Sec.	Sec.	Sec.	Poises / g/cm³
Olive oil	23	463.6	483.8	712.0	1.042
Car oil No. 8	23	313.9	317.6	471.1	.684
Transformer oil	21	122.7	123.3	185.9	.253
Floor oil	22	178.7	182.0	270.3	.383
Paraffin oil	20	684.4	692.2	1006.7	1.476
Three-fourths kerosene, one-fourth spindle	19	38.4	38.9	63.7	.0349
One-half kerosene, one-half spindle	20	45.3	45.3	74.2	.0587
One-fourth kerosene, three-fourths spindle	19	61.6	61.6	98.1	.1067
Spindle oil	21	88.8	90.8	139.0	.1774
Press oil No. 3	21	49.5	50.0	81.3	.0735
Meter oil	22	63.3	63.4	101.1	.1115
Old sucrose solution	17	253.9	254.2	399.9	.579
Fresh sucrose solution, 60 per cent	16	285.0	286.7	451.6	.656
Fresh sucrose solution, 40 per cent	17	46.9	46.6	76.8	.0642
Press oil No. 2	23	106.3	105.7	160.5	.2127
Press oil No. 4	22	137.3	135.6	205.6	.2838
Press oil No. 1	24	187.9	187.7	283.0	.403
Oil No. 885	23	183.3	185.2	274.8	.390
Oil X_2	24	278.8	274.9	413.1	.598
Spindle oil	25	83.5	84.0	129.9	.1622
Oil No. 458	26	251.5	250.3	380.7	.550
Oil X_5	27	336.1	336.2	501.8	.731
Glycerol solution, 52.2 per cent	20	.....	46.8	76.4	.0633
Glycerol solution, 70.0 per cent	20	.....	105.6	153.8	.2018
Glycerol solution, 80.3 per cent	20	212.9	211.9	328.0	.471
Glycerol solution, 85.0 per cent	20	.....	428.3	658.3	.962
Glycerol solution, 90.1 per cent	20	806.5	792.7	1231.1	1.806
Glycerol solution, 96.0 per cent	20	.....	1965.8	2950.6	4.336

[a] The times, Engler, for the glycerol solutions, were obtained by using an abnormal filling and a conversion factor, and are probably less reliable than the other readings.

The tests with sucrose solutions, while consistent enough with one another, would indicate a lower position of the calibration curve than obtained by tests with alcohol solutions. It is believed that this difference is due to the difference in surface tension, and that a calibrating liquid should be selected with a surface tension most nearly equal to that of oils, since Saybolt viscosimeters are mainly used for testing oils. Table 6 shows that 30 or 50 per cent ethyl alcohol solutions have a suitable surface tension.[16] On the other hand, it may be seen from Table 7 that the surface tension of sucrose solutions is much too high.[17]

[16] From Landolt-Börnstein, Physikalisch-chemische Tabellen, 4th ed., pp. 124, 128.

[17] Calculated from data of E. O. von Lippmann, Die Chemie der Zuckerarten, 2, p. 1119; 1904.

TABLE 6.—Surface Tension of Oils and Ethyl Alcohol Solutions, from Landolt-Börnstein

Liquid	Temperature at which density was taken	Density	Temperature at which surface tension was taken	Surface tension, in dynes per centimeter
	° C		° C	
Olive oil	20	0.910	20	31.74
		.917	20	34.5
		.9136	.....	32.09
		.9136	25.8	36.9
		.9136	54	30.56
Do	15	.989	15	35.6
			20	35.4
			25	34.9
			30	34.4
Russian illuminating oil	.....	.822	18	30.5
				32.3
American mineral oil	.....	.756	18	29.4
				30.4
Ethyl alcohol, 0 per cent	.....	.....	15	72.2
Ethyl alcohol, 10 per cent	.....	.....	15	51.2
Ethyl alcohol, 20 per cent	.....	.....	15	40.6
Ethyl alcohol, 30 per cent	.....	.....	15	34.7
Ethyl alcohol, 40 per cent	.....	.....	15	31.2
Ethyl alcohol, 50 per cent	.....	.....	15	29.1
Ethyl alcohol, 60 per cent	.....	.....	15	27.7

TABLE 7.—Surface Tension of Sucrose Solutions at 18.5° C (65.3° F) According to E. O. von Lippmann

Per cent sucrose	Surface tension in dynes per centimeter	Per cent sucrose	Surface tension in dynes per centimeter	Per cent sucrose	Surface tension in dynes per centimeter	Per cent sucrose	Surface tension in dynes per centimeter
0	72.8	20	67.8	40	63.4	60	59.4
5	71.5	25	66.7	45	62.4	65	58.4
10	70.3	30	65.7	50	61.4	70	57.5
15	69.1	35	64.5	55	60.4		

Since, for various reasons which have been given, distilled water, glycerol, and sucrose solutions are not suitable calibrating liquids for short-tube viscosimeters having a discharge into the air, tests with alcohol solutions must be given the most weight. Yet alcohol solutions, alone, are not sufficient to determine the values of A and B in equation (4), as the viscosities are too low; and if Higgins's method were used, an excessive extrapolation would be required. It is therefore necessary to find the constant A by other means, using Higgins's method for finding B, and for a rough check on the value of A.

8. A METHOD OF FINDING THE CONSTANT *A* IN EQUATION (4)

In order to obtain points on Fig. 1, it is necessary either to use liquids for which viscosities and densities are known for the temperatures employed, or to determine the kinematic viscosity by means of an instrument for which the constants *A* and *B* are known. After it had been found that the most viscous liquid which was available for calibrating purposes was a 50 per cent ethyl alcohol solution, and that no systematic difference in readings

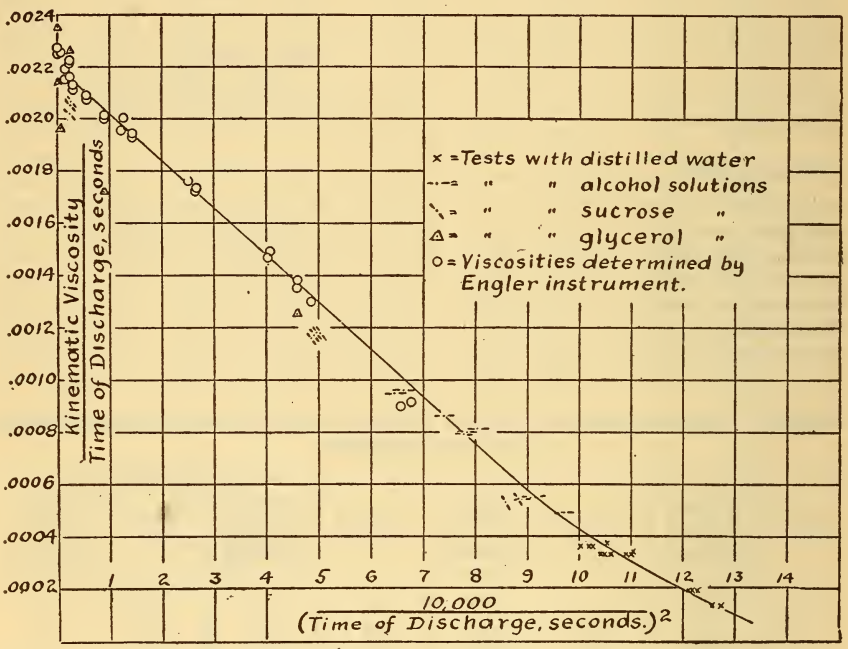

FIG. 1.—*Calibration curve of standard Saybolt Universal viscosimeter, by Higgins's method*

of the four Saybolt instruments could be detected, it was decided to use the method of viscosimeter comparison without temperature control [18] taking Saybolt instruments Nos. 110 and 111 and Engler No. 2204 U. Table 5 shows the results of these tests, and gives the kinematic viscosities calculated from the Engler time of discharge, using the equation.[19]

$$\frac{\mu}{\gamma} = 0.00147\, t - \frac{3.74}{t} \qquad (5)$$

[18] T. P. 100, p. 19.

[19] T. P. 100, p. 31. (This equation was derived from Fig. 4, and was not based on the tests with sucrose solutions shown in Fig. 2, which were few and discordant.)

If the kinetic energy correction (i. e., the last term) is negligible, and t_e and t_s are the times of discharge for the Engler and Saybolt viscosimeters, respectively, then

$$\frac{t_e}{t_s} = \frac{A}{0.00147} \tag{6}$$

A being the desired constant for the Saybolt instrument. Values of $\frac{t_e}{t_s}$, as calculated from data of Table 5, are shown in Fig. 2. The points for alcohol solutions are calculated from data in Table 4 taken in connection with Engler times of flow previously obtained.[20] As the kinetic energy correction decreases, the time ratio decreases

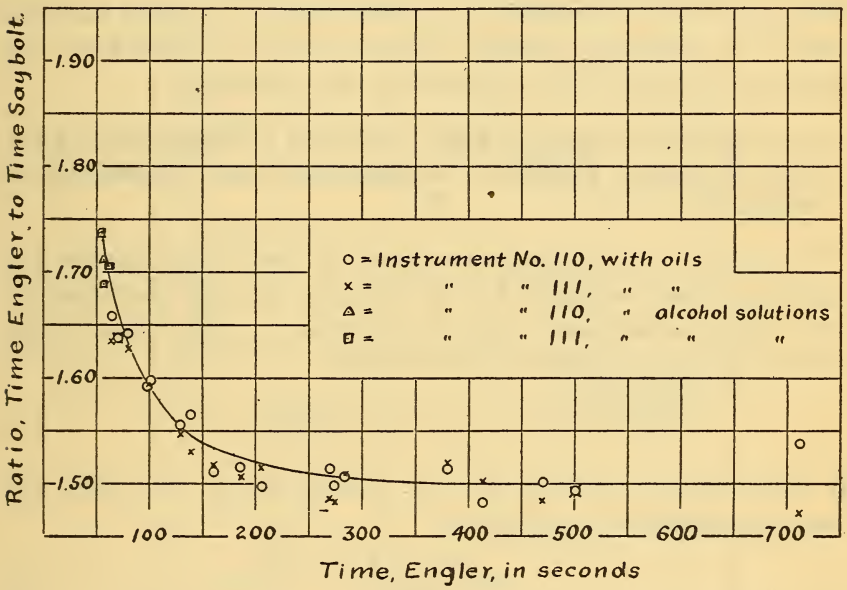

FIG. 2.—*Ratio of times of discharge of Engler viscosimeter No. 2204 U and the standard Saybolt Universal viscosimeter*

until, when the kinetic energy correction is negligible, the ratio becomes a constant. Thus the lowest ratio obtained should be the constant value required. The most viscous oil used gave a time ratio of 1.472 and 1.455, respectively, for instruments Nos. 110 and 111. These values are probably too low on account of experimental error. On the other hand, the average of 12 oils of high viscosity gives 1.503 and 1.495, which would be slightly too high if the kinetic energy correction was not negligible in all cases. If the average value 1.499 is used, then from equation (6), $A = 0.00220$.

[20] T. P. 100, p. 24.

9. DETERMINATION OF THE CONSTANT *B* BY HIGGINS'S METHOD

The value of A, as determined above, fixes one end of the calibration curve of Fig. 1, which must pass through the point with an ordinate of 0.00220 and an abscissa of zero. The rest of the curve has been located by the tests with distilled water and alcohol solutions of Table 4 and the tests of Table 5. It is seen that the line obtained is nearly parallel to the one indicated by the tests with sucrose solutions, and that the tangent of the acute angle formed by this line with the axis of abscissas, which determined the value of B, is about 1.8.

This value of B, and that of A given above, were found without reference to the dimensions of the instruments. A check of these values by Herschel's method will serve also as a check on the accuracy with which the dimensions were measured.

10. DETERMINATION OF THE COUETTE CORRECTION AND THE KINETIC ENERGY CORRECTION BY HERSCHEL'S METHOD

By substituting the dimensions given in Table 2 in equation (3) equations may be found for the different Saybolt instruments. This was done for instruments Nos. 108 to 111, and the average for the four instruments was found to be

$$\frac{\mu}{\gamma}(1.229+\lambda)=0.00289t-\frac{2.387}{t}m. \tag{7}$$

If the Couette correction and the kinetic energy correction are both disregarded in equation (3),

$$\mu^1=\frac{\pi\, g d^4 \gamma\, h\, t}{128\, Q\, l}. \tag{8}$$

Then if "Reynolds's criterion," $\frac{v\, d\, \gamma}{\mu}$ is plotted against $\frac{\mu^1}{\mu}$, it may be shown that [21]

$$\lambda=\left(\frac{\mu^1}{\mu}-1\right)l. \tag{9}$$

$$m=\frac{32\, l}{\tan\theta\, d}. \tag{10}$$

The value of $\frac{\mu^1}{\mu}$ corresponding to a value of Reynolds's criterion of zero must be used in equation (9); that is, $\frac{\mu^1}{\mu}$ must be found

[21] T. P. 100, p. 13.

by prolonging the calibration curve until it intersects the axis of abscissas. In equation (10) θ is the angle between the calibration curve and the axis of abscissas.

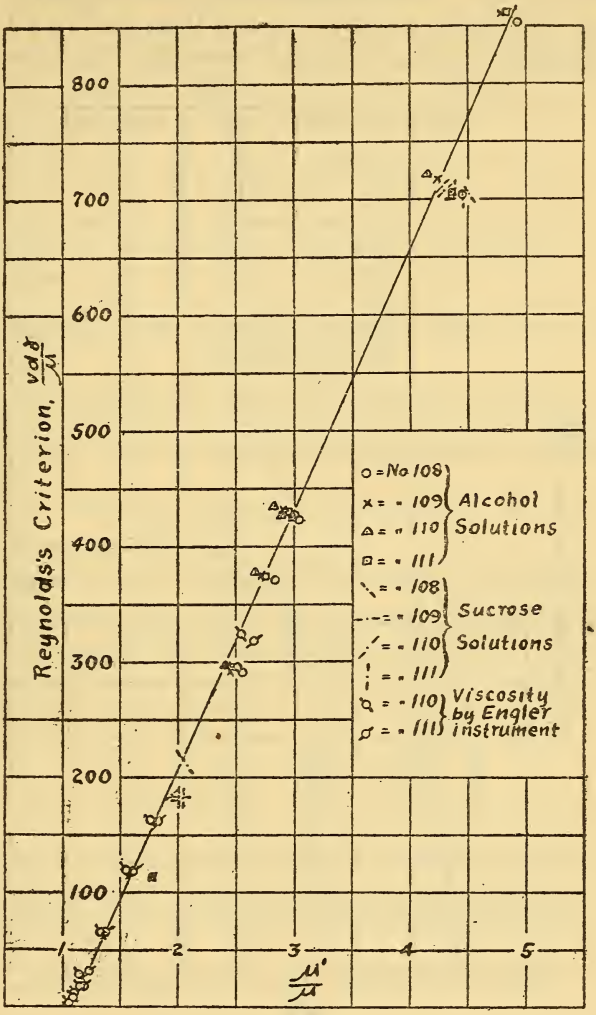

FIG. 3.—*Calibration curve of Standard Saybolt Universal viscosimeter, by Herschel's method.*

Fig. 3 was calculated from the data of Tables 4 and 5, while Fig. 4 gives an enlarged view of the lower end of the calibration curve for use in finding the Couette correction. By extrapolation, $\frac{\mu^1}{\mu}$ has a value of 1.044 for instrument No. 110, and 1.090 for instrument No. 111, giving an average of 1.067. These values,

if obtained approximately, merely by averaging the nine lowest values obtained by experiment, would have been 1.058, 1.104, and 1.081, respectively. These latter results are, of course, too high, as Reynolds's criterion must be greater than zero in all tests.

From equation (9) the Couette correction is 0.067 (1.229) =

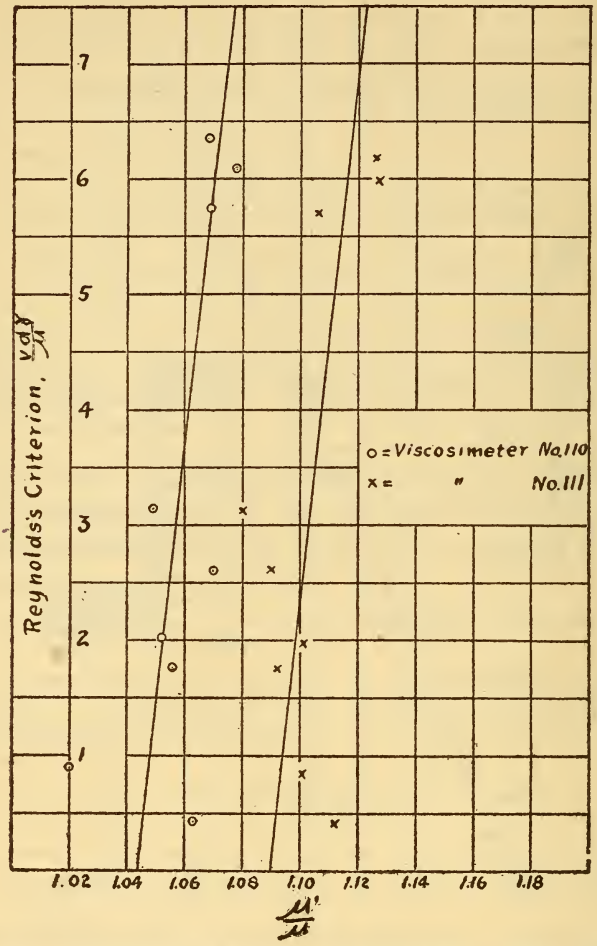

FIG. 4.—*Enlarged view of bottom of Fig. 3, used to determine the Couette correction*

0.0824, or, since it is usually expressed as proportional to the diameter, $\lambda = 0.466\ d$. When the kinetic energy correction is negligible, combining equations (4) and (7) gives

$$\frac{\mu}{\gamma} = A\ t = \frac{0.00289}{(1.229+\lambda)} t \qquad (11)$$

from which, with λ equal to 0.082, the value of A is found to be 0.00220.

From Fig. 3 tan θ is 225, so that by equation (10) $m = 0.989$. By combining the last terms of equations (4) and (7)

$$\text{kinetic energy correction} = \frac{B}{t} = \frac{2.387\ m}{(l+\lambda)\ t} \qquad (12)$$

from which, with $m = 0.989$ and $\lambda = 0.082$, the value of B becomes 1.801. The complete equation for determining the kinematic viscosity from the time of discharge is therefore

$$\frac{\mu}{\gamma} = 0.00220\ t - \frac{1.80}{t} \qquad (13)$$

which applies to a Saybolt Universal viscosimeter of normal dimensions.

11. EQUATIONS FOR INSTRUMENTS NOT OF NORMAL DIMENSIONS

According to Ubbelohde, the kinematic viscosity may be determined from the time of discharge of the Engler viscosimeter by the equation

$$\frac{\mu}{\gamma} = \left(4.072\frac{t}{t_w} - 3.513\frac{t_w}{t}\right) 0.01797 \qquad (14)$$

where t_w is the time of discharge for 200 cm³ of water at 20° C. (68° F.), commonly called the water rate. Comparing equations (4) and (13) it is seen that Ubbelohde assumes that A is inversely and B is directly proportional to the water rate. Now, in reality [22]

$$t_w = \frac{\frac{\mu}{\gamma}}{2A}\left(1 + \sqrt{1 + \frac{4AB}{\left(\frac{\mu}{\gamma}\right)^2}}\right) \qquad (15)$$

so that there is little justification for Ubbelohde's assumptions. It therefore seems preferable to fall back on equation (3), and to assume that the slight variations in dimensions allowed in Table 3 will not cause any change in the values of m and of $\frac{\lambda}{d}$. This assumption has some justification in the slight difference between the values of these quantities, as found with instruments Nos. 108 to 111 and with Nos. 580 and 727. Taking, accordingly, the values of $\frac{\lambda}{d}$ and m, as found above, and the limiting viscosimeter dimensions from Table 3, limiting equations may be found for instruments of standard, though not of the average or normal dimensions. Equation (16) was calculated by selecting the

[22] Proc., A. S. T. M., 15 (1), p. 318; 1915.

dimensions which would tend to give a minimum time of discharge while equation (17) was calculated from dimensions to give a maximum time.

$$\frac{\mu}{\gamma}=0.00234\,t-\frac{1.82}{t}\ \text{(minimum)} \tag{16}$$

$$\frac{\mu}{\gamma}=0.00204\,t-\frac{1.79}{t}\ \text{(maximum)} \tag{17}$$

12. APPLICATIONS OF THE NEW EQUATIONS

Table 8 has been calculated from equations (13), (16), and (17).

TABLE 8.—Times of Discharge for Standard Saybolt Universal Viscosimeters

Liquid	Temperature	Kinematic viscosity	Time of discharge		
			Minimum	Normal	Maximum
	°C		Sec.	Sec.	Sec.
30 per cent ethyl alcohol	20	0.02841	34.6	35.7	37.4
Do	25	.02292	33.2	34.2	35.7
50 per cent ethyl alcohol	20	.03144	35.4	36.6	38.4
Do	25	.02636	34.2	35.2	36.8
A heavy oil		1.00	428.0	455.0	490.0

It should be noted that the table is based on the assumption that the departure of each dimension from the normal was of such sign as to produce the maximum possible deviation from the normal time of flow. In practice, however, the departures would tend to counteract one another, so that the variations in times of discharge would not be as great as shown in Table 8. The heavy oil has been assumed of a high enough viscosity to make the kinetic energy correction negligible. Water is not recommended for a calibrating liquid, and the time of discharge for water has accordingly been omitted.

Equations for calculating kinematic viscosity serve as a ready means for calculating conversion tables between times of discharge of different types of viscosimeters. Thus, equations (5) and (13) have been used to calculate Table 9 for conversions between the Saybolt Universal viscosimeter of normal dimensions and Engler viscosimeter No. 2204 U. All previously published conversion tables or diagrams for Saybolt Universal viscosimeters are based on tests with instruments of obsolete dimensions and should therefore be discarded.[23]

[23] T. P. 100, p. 33; T. B. Stillman, Examination of Lubricating Oils, p. 22, 1914; R. F. Bacon and W. A. Hamor, The American Petroleum Industry, 1, p. 111, 1916.

TABLE 9.—Conversion Table for Viscosimeters

[Engler No. 2204 U and standard Saybolt Universal]

Time, Engler	Time, Saybolt	Time, ratio	Engler degrees = time, Engler / 51.3[a]	Kinematic viscosity	Time, Engler	Time, Saybolt	Time, ratio	Engler degrees = time, Engler / 51.3[a]	Kinematic viscosity
Sec.	Sec.			Poises / g/cm³	Sec.	Sec.			
56	32.4	1.73	1.09	0.0155	130	83.6	1.56	2.54	0.1624
58	33.7	1.72	1.13	.0208	140	90.6	1.55	2.73	.1793
60	35.1	1.71	1.17	.0259	150	97.4	1.54	2.93	.1956
62	36.5	1.70	1.21	.0308	160	104.4	1.53	3.12	.2121
64	37.8	1.69	1.25	.0356	180	117.7	1.53	3.51	.2437
66	39.2	1.68	1.29	.0403	200	131.5	1.52	3.90	.2753
68	40.6	1.68	1.33	.0448	225	148.3	1.52	4.39	.3140
70	42.0	1.67	1.36	.0495	250	165.3	1.51	4.88	.3523
75	45.5	1.65	1.46	.0603	275	182.0	1.51	5.36	.3904
80	49.0	1.63	1.56	.0709	300	199.0	1.51	5.85	.4282
85	52.4	1.62	1.66	.0810	325	215.6	1.51	6.34	.4660
90	55.8	1.61	1.76	.0905	350	232.9	1.50	6.82	.5038
95	59.4	1.60	1.85	.1003	375	249.2	1.50	7.31	.5413
100	62.8	1.59	1.95	.1095	400	266.1	1.50	7.80	.5784
110	69.8	1.58	2.15	.1278	500	333.0	1.50	9.75	.7271
120	76.8	1.56	2.34	.1453	600	400.0	[b] 1.50	11.70	.8753

[a] See T. P. 100, p. 23.
[b] This value holds good for all higher viscosities.

13. THE REDWOOD VISCOSIMETER

It is sometimes necessary for the export trade to consider the Redwood viscosimeter. This instrument has never been standardized with respect to its dimensions, and the standardization with rapeseed oil is unsatisfactory,[24] as it is equivalent to fixing the value of A in equation (4) and leaving B undetermined. Therefore all equations for the Redwood instrument [25] must be regarded as approximations, and it is probable that these instruments would not give as concordant results with liquids of low viscosity as with the rapeseed oil with which a standard time of discharge is obtained.

If Higgins's equation is taken as probably as good as any, and equation (5) is used for the Engler viscosimeter, Table 10 may be obtained to supplement Table 9.

[24] L. Archbutt and R. M. Deeley, Lubrication and Lubricants, p. 168, 1912.
[25] T. P. 100, p. 31.

TABLE 10.—Conversion Table for Redwood, Saybolt, and Engler Viscosimeters

Time, Engler	Time, ratio			Time, Redwood	Time, Engler	Time, ratio			Time, Redwood
	Engler-Saybolt	Engler-Redwood	Saybolt-Redwood[a]			Engler-Saybolt	Engler-Redwood	Saybolt-Redwood[a]	
Sec.				Sec.	Sec.				Sec.
56	1.73	2.61	1.51	21.5	140	1.55	1.81	1.17	77.5
58	1.72	1.93	1.12	30.0	150	1.54	1.80	1.17	83.2
60	1.71	1.93	1.13	31.2	160	1.53	1.80	1.17	89.1
62	1.70	1.92	1.13	32.2	180	1.53	1.80	1.17	100.3
64	1.69	1.91	1.13	33.4	200	1.52	1.79	1.18	111.9
66	1.68	1.91	1.13	34.6	225	1.52	1.79	1.18	126.0
68	1.68	1.90	1.14	35.8	250	1.51	1.78	1.18	140.3
70	1.67	1.90	1.14	36.9	275	1.51	1.78	1.18	154.4
75	1.65	1.88	1.14	39.8	300	1.51	1.78	1.18	168.5
80	1.63	1.87	1.15	42.7	325	1.51	1.78	1.18	183.0
85	1.62	1.86	1.15	45.7	350	1.50	1.78	1.18	197.0
90	1.61	1.86	1.15	48.5	375	1.50	1.77	1.18	211.3
95	1.60	1.85	1.16	51.4	400	1.50	1.77	1.18	225.5
100	1.59	1.84	1.16	54.3	500	1.50	1.77	1.18	282.0
110	1.58	1.83	1.16	60.1	600	1.50	1.77	1.18	339.0
120	1.56	1.82	1.16	66.0					
130	1.56	1.81	1.17	71.7					

[a] These values were calculated directly from the times of discharge and differ in some cases by 0.01 from values calculated from the two previous columns.

14. CONCLUSION

Table 3 and equations (13), (16), and (17) give the complete standardization of the Saybolt Universal viscosimeter. It has previously been impossible to determine whether a Saybolt Universal viscosimeter gave normal readings, as neither the dimensions nor normal times of flow for any given liquids were known. Now that these data have been determined, limit gages have been prepared, and the Bureau of Standards is now in a position to certify whether or not a given instrument is of standard dimensions.

WASHINGTON, February 3, 1919.

APPENDIX

THE VISCOSITY OF GLYCEROL SOLUTIONS

Archbutt and Deeley give viscosities of glycerol solutions of various specific gravities. Different experimenters do not agree very closely in regard to the relation between the density and the concentration of glycerol solutions. Table 11 is based on data of Gerlach [26] and of Archbutt and Deeley.

TABLE I.—Densities and Viscosities of Glycerol Solutions at 20° C (68° F) According to Archbutt and Deeley and Gerlach

Per cent glycerol	Density in grams per cubic centimeter	Viscosity in poises	Kinematic viscosity	Per cent glycerol	Density in grams per cubic centimeter	Viscosity in poises	Kinematic viscosity
5	1.0098	0.01181	0.01170	55	1.1393	0.07664	0.06727
10	1.0217	.01364	.01335	60	1.1528	.1031	.08943
15	1.0337	.01580	.01529	65	1.1662	.1451	.1244
20	1.0461	.01846	.01765	70	1.1797	.2149	.1822
25	1.0590	.02176	.02055	75	1.1932	.3371	.2825
30	1.0720	.02585	.02411	80	1.2066	.5534	.4586
35	1.0855	.03115	.02870	85	1.2201	1.025	.8401
40	1.0989	.03791	.03450	90	1.2335	2.076	1.683
45	1.1124	.04692	.04218	95	1.2465	4.801	3.852
50	1.1258	.05908	.05248				

To avoid the effect of possible error in Gerlach's relation between density and concentration, solutions were made up to approximately the desired concentration by the above table, and then the viscosities of the solutions were interpolated from the table after the exact density had been found.

It was noted that the 80 per cent solution evaporated, while the 90 per cent solution absorbed moisture. Consequently, if it were not for the doubt as to the accuracy of Archbutt and Deeley's values of the viscosity, which appear to be especially erratic for concentrations in this neighborhood, solutions of about 85 per cent would be the most valuable for calibrating purposes, because the viscosity would remain most constant when exposed to the air in technical viscosimeters.

A much greater range of viscosities may be obtained with glycerol than with alcohol or sucrose solutions. Comparing kinematic viscosities at 20° C (68° F), the maximum which can be obtained with ethyl alcohol is 0.03144 with a 50 per cent solution, while practically the maximum obtainable with sucrose is 0.4392 for a 60 per cent solution. That a 60 per cent solution has approximately the greatest concentration which can be employed is shown by a table given by von Lippmann [27] for the per cent of sucrose in saturated solutions at various temperatures. This shows an increase from 64.18 per cent at 0° C (32° F) to 82.97 per cent at 100° C (212° F), the value at 20° C (68° F) being 67.09 per cent.

2. THE VISCOSITY OF CASTOR OIL

On account of the uncertainty in regard to the viscosity of glycerol solutions and the rapid change of viscosity with the concentration, it is probable that the best calibrating liquid for very high viscosities is castor oil, although this can not be regarded as

[26] G. T. Gerlach, Chemische Industrie 7, p. 277; 1884.

[27] E. O. von Lippmann, Chemie der Zuckerarten, 2, p. 1089; 1904.

entirely satisfactory on account of slight differences between different samples. However, the difficulty of accurate determinations of the viscosity of such a very viscous liquid is so great that a viscosity determination may well be considered as a test of the skill of the experimenter rather than as a test of the variation between samples. The extent of this variation may be judged from the work of Deering and Redwood [28] on 23 samples of Indian oil. Their results were rearranged in the order of the density, but no regular variation of viscosity with the density could be observed. Table II shows the maximum and minimum values for the 23 samples.

TABLE II.—Density and Viscosity of Castor Oil According to Deering and Redwood

Specific gravity at 60° F (15.6° C)	Density at 60° F	Time of discharge, in seconds, Redwood viscosimeter, at 100° F (37.8° C)	Kinematic viscosity at 100° F
0.9637 minimum	0.9628	1190 maximum	3.093
0.9639	.9630	1160 minimum	3.015
0.9642 maximum	.9638	1174	3.051

The kinematic viscosities were determined by Higgins's equation.[29]

$$\frac{\mu}{\gamma}=0.00260\ t-\frac{1.715}{t}. \qquad (18)$$

A slight error might be caused by deterioration of the oil with time. Lewkowitsch [30] says:

"Properly refined castor oil keeps very well and does not easily turn rancid, as observations made in the author's laboratory have shown. A sample exposed to the atmosphere for four years contained only 1 per cent of free fatty acids."

Another or possibly the same sample, which was exposed for four years, had its specific gravity at 60° F (15.6° C) increased from 0.9591 to 0.9629.

Probably the most accurate determinations of the viscosity of castor oil have been made by Kahlbaum and Räber.[31] Their results are given in Table III, to which the kinematic viscosities have been added.

TABLE III.—Density and Viscosity of Castor Oil According to Kahlbaum and Räber

Temperature in degrees centigrade	Density in grams per cubic centimeter	Viscosity in poises	Kinematic viscosity	Temperature in degrees centigrade	Density in grams per cubic centimeter	Viscosity in poises	Kinematic viscosity
5	0.9707	37.60	38.74	23	.9583	7.67	8.00
6	.9700	34.475	35.54	24	.9576	7.06	7.37
7	.9693	31.56	32.56	25	.9569	6.51	6.80
8	.9686	28.90	29.84	26	.9562	6.04	6.32
9	.9679	26.45	27.33	27	.9555	5.61	5.87
10	.9672	24.18	25.00	28	.9548	5.21	5.46
11	.9665	22.075	22.84	29	.9541	4.85	5.08
12	.9659	20.075	20.78	30	.9534	4.51	4.73
13	.9652	18.25	18.91	31	.9527	4.21	4.42
14	.9645	16.61	17.22	32	.9520	3.94	4.14
15	.9638	15.14	15.71	33	.9513	3.65	3.84
16	.9631	13.805	14.33	34	.9506	3.40	3.58
17	.9624	12.65	13.14	35	.9499	3.16	3.33
18	.9617	11.625	12.09	36	.9492	2.94	3.10
19	.9610	10.71	11.15	37	.9485	2.74	2.89
20	.9603	9.86	10.27	38	.9478	2.58	2.72
21	.9596	9.06	9.44	39	.9471	2.44	2.58
22	.9589	8.34	8.70	40	.9464	2.31	2.44

[28] W. H. Deering and Boverton Redwood, Jour. Soc. for Chem. Industriy **13**, p. 959; 1894.

[29] W. F. Higgins, Collected Researches, Nat. Phy. Lab., **11**, p. 12, 1914.

[30] J. Lewkowitsch, Chemical Technology and Analysis of Oils, Fats, and Waxes, **2**, pp. 395–396; 1915.

[31] W. A. Kahlbaum and S. Räber, Abd. der Kaiserl. Leop.-Carol. Deutschen Akademie der Naturforscher **84**, p. 201; 1905.

Castor oil has the advantage, as compared with glycerol and sucrose solutions, of evaporating much less easily, and hence it may be used over a much greater range of temperatures. Table IV gives viscosities as determined by Archbutt and Deeley,[32] together with the kinematic viscosities which have been calculated from their data.

TABLE IV.—Density and Viscosity of Castor Oil According to Archbutt and Deeley

Temperature		Viscosity in poises	Density in grams per cubic centimeter	Kinematic viscosity
Degrees Fahrenheit	Degrees centigrade			
100	37.8	2.729	0.9473	2.881
150	65.6	.605	.9284	.6517
212	100.0	.169	.9050	.1867

By interpolation in Table II the density according to Kahlbaum and Räber would be 0.9634 at 15.6° C, which may be compared with Redwood and Deering's values in Table I, while the density at 37.8° C would be 0.9479, which agrees fairly well with Archbutt and Deeley's value. Similarly, Kahlbaum and Räber give a kinematic viscosity of 2.75 at 37.8° C, which may be compared with the values obtained by the other experimenters. It would appear that while the variation in density and viscosity is appreciable, it is too small to be of importance with short-tube technical instruments.

[32] L. Archbutt and R. M. Deeley, Lubrication and Lubricants, pp. 186–188; 1912.